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Radiative MHD Boundary Layer Flow of A Polar Fluid Through A Porous Medium With Couple Stress And Rotational Parameter On Horizontal Plate

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ABSTRACT: *This paper examines the problem of MHD unsteady flow of a polar fluid through a porous medium of an electrically conducting fluid bounded by an infinite horizontal and porous plate. Expressions for the velocity and temperature field is derived and the effects of the different parameters entering into the problem are graphically presented and discussed. It is bring observed that the Nusselt number increases with increase in Prandtl number as well as velocity slip parameter.*

KEY WORDS: *Free stream velocity, Magnetic field, Polar fluid, Porous medium,* radiation, Rotational velocity.

I. INTRODUCTION

The study of the flow through a porous medium is of great importance of geophysicists and fluid dynamicists. Brinkman [1]. Raptis et al. [2], Yamomota and Yoshida [3] have studied flow through a porous medium considering generalized Darcy's law. In all above research paper generalized Darcy's law is derived without taking into account the angular velocity of the fluid particles. These fluids are known as polar fluids in the literature. Aero et al. [4], Raptis [5] derived flow equations of such fluids taking angular velocity into consideration. Recently Jain et al. [6], Jain and Gutpa [7], Taneja et al. [8] considered magnetic effects on polar flow. Polar fluids belong to a class of fluids with microstructure and have asymmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium. Some authors viz. Ahmadi [9], Kim [10] used the word micropolar in place of polar because of the microstructure property of the fluid particles.

Radiative convective flows are having much application in evaporations from large open water reservoirs, solar power technology and space vehicle recentry. England and Emery [11], Soundalgekar and Takhar [12], Raptis and Perdikis [13] have studied the effects of thermal radiation in their problems. Recently Chandrakala and Antony Raj [14] have considered mass transfer effects along with radiative heat on a moving vertical plate in presence of magnetic field.

The object of this paper is the study of the unsteady flow of an electrically conducting, radiating and polar fluid through a porous medium bounded by an infinite horizontal and porous plate with constant suction by the presence of a magnetic field when the free stream velocity is constant.

II. MATHEMATICAL FORMULATION AND SOLUTIONS OF THE PROBLEM

We consider the unsteady, Radiative flow of a polar fluid through a porous medium bounded by an infinite horizontal and permeable plate. The x-axis is taken along the plate and the y-axis normal to it. The fluid is assumed to be viscous, incompressible and electrically conducting. The magnetic field of uniform strength is assumed to be applied parallel to y-axis. The flow is assumed to be at small magnetic Reynolds number which enables us to neglect the induced magnetic filed.

Under these assumptions, the physical variables are function of y except the pressure p. Hence the flow field is governed by the following equations. Under the above stated assumptions the equations of continuity, linear momentum, angular momentum and energy are :

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + (\upsilon + \upsilon_r) + \frac{\partial^2 u}{\partial y^2} + 2\upsilon_r\frac{\partial \omega}{\partial y} - \frac{\upsilon u}{K(t)} - \frac{\sigma\mu_e B^2 u}{\rho} \tag{2.2}$$

$$\frac{\partial \omega}{\partial t} + \nu \frac{\partial \omega}{\partial y} = \frac{\gamma}{I} \frac{\partial^2 \omega}{\partial y^2} \tag{2.3}$$

$$\frac{\partial T}{\partial t} + \nu \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{dq_r}{dy} \tag{2.4}$$

here ρ denotes the density, u the velocity along the x-direction and ν the velocity along the y-direction, υ_r is the kinematic rotational viscosity, ω the mean angular velocity of rotation of the particles, K(t) the permeability of the porous medium, σ the electrical conductivity, B the magnetic induction, I a scalar constant of dimension equal to that of the moment of inertia of unit mass and

$$\gamma = C_a + C_d$$

where C_a and C_d are coefficient of couple stress viscosities. Remaining symbols have their usual meaning.
The corresponding boundary conditions as :

$$u = L_1 \frac{\partial u}{\partial y}, \qquad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \qquad T = T_w, \ \text{at } y = 0$$

$$u \to U(t) = U_0(1 + \in e^{-nt}), \ \omega \to 0, T \to T_\infty, \text{ as } y \to \infty \tag{2.5}$$

where $L_1 = \left(\frac{2 - m_1}{m_1}\right) L$, L being mean free path and m_1 the Maxwell's reflection coefficient.

The permeability of porous medium is assumed to be of the form

$K(t) = K_0 (1+A \in e^{-nt})$

Where K_0 is the mean permeability of the medium, A and n are real positive constant, t the time and ∈ is small such that ∈A <<1.

Integration of equation (2.1) for constant suction gives,

$$\nu = \nu_0 \tag{2.6}$$

The radiative heat flux q_r is considered as

$$\frac{dq_r}{dy} = 4(T - T_\infty) I_1 \qquad (2.7$$

where $I_1 = \int_0^\infty K_{\lambda n} \frac{\partial e_{b\lambda}}{\partial T} d\lambda$,

For the free stream velocity the equation (2.2) gives

$$\frac{\partial U}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial x} - \left(\frac{\nu}{K(t)} + \frac{\sigma \mu_e B^2}{\rho} \right) U_\infty \tag{2.8}$$

Introducing the following non-dimensional quantities

$$u^* = \frac{u}{U_0}, \qquad y^* = \frac{y \nu_0}{\upsilon}, \qquad \omega^* = \frac{\omega \upsilon}{\nu_0 U_0}, \qquad t^* = \frac{\nu_0^2 t}{\upsilon},$$

$$n^* = \frac{\upsilon n}{\nu_0^2}, \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$K^* = \frac{K \nu_0^2}{\upsilon^2}$ (Permeability parameter), $\alpha = \frac{\upsilon_r}{\upsilon}$ (Rotational parameter)

$\beta = \dfrac{I\upsilon}{\gamma}$ (Couple stress parameter), $M^2 = \dfrac{\sigma\mu_e B^2 \upsilon}{\rho \nu_0^2}$ (Magnetic field parameter)

$Pr = \dfrac{\mu C_p}{k}$ (Prandtl number), $S = \dfrac{4\upsilon I_1}{\rho C_p \nu_0^2}$ (Radiation parameter)

$h_1 = \dfrac{L_1 \nu_0}{\upsilon}$ (Velocity slip parameter)

We get in view of equations (2.6), (2.7) and for free stream velocity, the equations of motion in non-dimensional form after dropping the asterisks over them, reduced to :

$$(1+\alpha)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} + 2\alpha\frac{\partial \omega}{\partial y} - \left[M^2 + \frac{1}{K_0(1 + A \in e^{-nt})}\right](u - 1 - \in e^{-nt}) - \frac{\partial u}{\partial t} = -\frac{d}{dt}(1 + \in e^{-nt}) \tag{2.9}$$

$$\frac{\partial^2 \omega}{\partial y^2} + \beta\frac{\partial \omega}{\partial y} = \beta\frac{\partial \omega}{\partial t} \tag{2.10}$$

$$\frac{\partial^2 \theta}{\partial y^2} + Pr\frac{\partial \theta}{\partial y} - Pr\, S\,\theta = Pr\frac{\partial \theta}{\partial t} \tag{2.11}$$

With corresponding boundary conditions as

$$u = h_1\frac{\partial u}{\partial y}, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad \theta = 1 \quad \text{at } y = 0$$

$$u = 1 + \in e^{-nt}, \quad \omega \to 0,\ \theta \to 0, \text{ as } y \to \infty \tag{2.12}$$

Solving the equations (2.9) to (2.11), we assume

$u(y, t) = u_0(y) + \in e^{-nt} u_1(y)$

$\omega(y, t) = \omega_0(y) + \in e^{-nt} \omega_1(y)$

$\theta(y, t) = \theta_0(y) + \in e^{-nt} \theta_1(y)$ (2.13)

We get the solutions, after neglecting the coefficient of $\in^2$ and using corresponding boundary conditions as :

$$u = 1 + \in e^{-nt} + C_3 e^{-R_2 y} - \frac{2C_1\alpha\beta}{(R_1+\beta)(R_2-\beta)}e^{-\beta y} + \in e^{-nt}\left[C_4 e^{-R_4 y} - \frac{2C_2\alpha m_1}{(R_3+m_1)(R_4-m_1)}e^{-m_1 y} + \frac{AC_3}{K_0(R_2+R_3)(R_4-R_2)}e^{-R_2 y} - \frac{2AC_1\alpha\beta}{K_0(R_1+\beta)(R_2-\beta)(R_3+\beta)(R_4-\beta)}e^{-R_2 y}\right] \tag{2.14}$$

$$\omega = C_1 e^{-\beta y} + \in e^{-nt} C_2 e^{-m_1 y} \tag{2.15}$$

$$\theta = e^{-R_5 y} \tag{2.16}$$

Where

$R_1 = \dfrac{-1 + \sqrt{1 + 4(1+\alpha)N^2}}{2(1+\alpha)}$, $\qquad R_2 = \dfrac{1 + \sqrt{1 + 4(1+\alpha)N^2}}{2(1+\alpha)}$

$R_3 = \dfrac{-1 + \sqrt{1 + 4(1+\alpha)(N^2 - n)}}{2(1+\alpha)}$, $\qquad R_4 = \dfrac{1 + \sqrt{1 + 4(1+\alpha)(N^2 - n)}}{2(1+\alpha)}$

$$R_5 = \frac{\Pr + \sqrt{\Pr^2 + 4\Pr(S - n)}}{2}, \qquad N^2 = M^2 + \frac{1}{K_0}$$

$$m_1 = \frac{\beta + \sqrt{\beta^2 - 4n\beta}}{2},$$

$$C_1 = \frac{R_2^2(R_1+\beta)(R_2-\beta)}{\beta\left[2\alpha\left\{R_2^2(1+h_1\beta) - \beta^2(1+h_1R_2)\right\} - (1+h_1R_2)(R_1+\beta)(R_2-\beta)\right]}$$

$$C_2 = \frac{B_2C_3\left\{R_4^4(1+h_1R_2) + R_2^2(1+h_1R_4)\right\} + B_3C_1\left\{R_4^2(1+h_1\beta) - \beta^2(1+h_1R_4)\right\} - R_4^2}{m_1(1+h_1R_4) + B_1\left\{m_1^2(1+h_1R_4) - R_4^2(1+h_1m_1)\right\}}$$

$$C_3 = \frac{1}{1+h_1R_2}\left[\frac{2C_1\alpha\beta(1+h_1\beta)}{(R_1+\beta)(R_2-\beta)} - 1\right], C_4 = \frac{1}{1+h_1R_4}\left[\frac{2C_2\alpha m_1(1+h_1m_1)}{(R_3+m_1)(R_4-m_1)} - \frac{AC_3(1+h_1R_2)}{K_0(R_2+R_3)(R_4-R_2)}\right.$$

$$\left. + \frac{2AC_1\alpha\beta(1+h_1\beta)}{K_0(R_1+\beta)(R_2-\beta)(R_3+\beta)(R_4-\beta)} - 1\right]$$

$$B_1 = \frac{2\alpha m_1}{(R_3+m_1)(R_4-m_1)}, \quad B_2 = \frac{A}{K_0(R_2+R_3)(R_4-R_2)}, B_3 = \frac{2A\alpha\beta}{K_0(R_1+\beta)(R_2-\beta)(R_3+\beta)(R_4-\beta)}$$

III. SKIN FRICTION AND RATE OF HEAT TRANSFER

Once the expressions for velocities and temperature are known, it is important to calculate the skin-friction and Nusselt number.

The skin-friction due to velocity is given, in non-dimensional form after dropping the asterisk

$$\tau = \frac{\tau_w}{\rho U_0 v_0}$$

$$= (1+\alpha)\left[-C_3R_2 + \frac{2C_1\alpha\beta^2}{(R_1+\beta)(R_2-\beta)} + \in e^{nt}\left\{-C_4R_4 + \frac{2C_2\alpha m_1^2}{(R_3+m_1)(R_4-m_1)} - \right.\right.$$

$$\left.\left.\frac{AC_3R_2}{K_0(R_2+R_3)(R_4-R_2)} + \frac{2AC_1\alpha\beta^2}{K_0(R_1+\beta)(R_2-\beta)(R_3+\beta)(R_4-\beta)}\right\}\right] \tag{3.1}$$

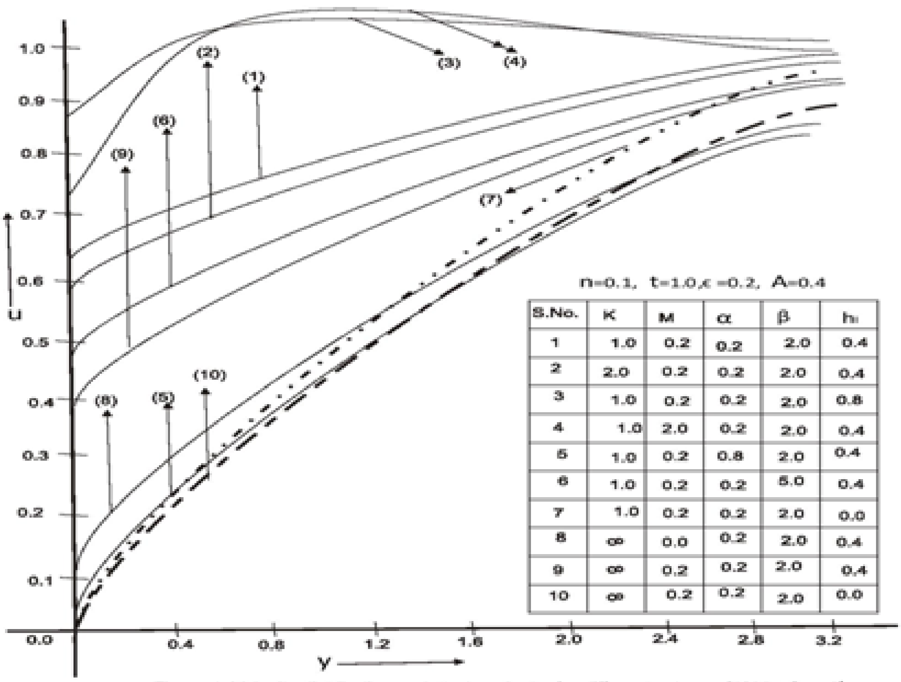
and the rate of heat transfer is given by

$$Nu = \frac{-L_1}{(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

in non-dimensional form after dropping the asterisk

$$Nu = h_1R_5 \tag{3.2}$$

IV. FIGURES AND TABLES

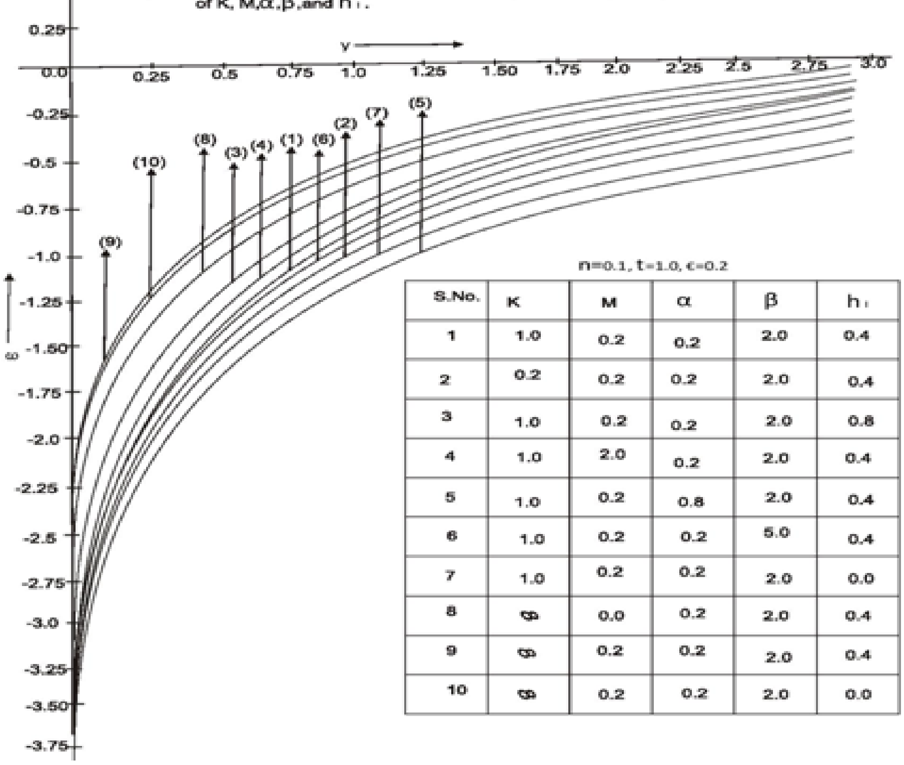


n=0.1, t=1.0, ϵ=0.2, A=0.4

S.No.	K	M	α	β	h_1
1	1.0	0.2	0.2	2.0	0.4
2	2.0	0.2	0.2	2.0	0.4
3	1.0	0.2	0.2	2.0	0.8
4	1.0	2.0	0.2	2.0	0.4
5	1.0	0.2	0.8	2.0	0.4
6	1.0	0.2	0.2	5.0	0.4
7	1.0	0.2	0.2	2.0	0.0
8	∞	0.0	0.2	2.0	0.4
9	∞	0.2	0.2	2.0	0.4
10	∞	0.2	0.2	2.0	0.0

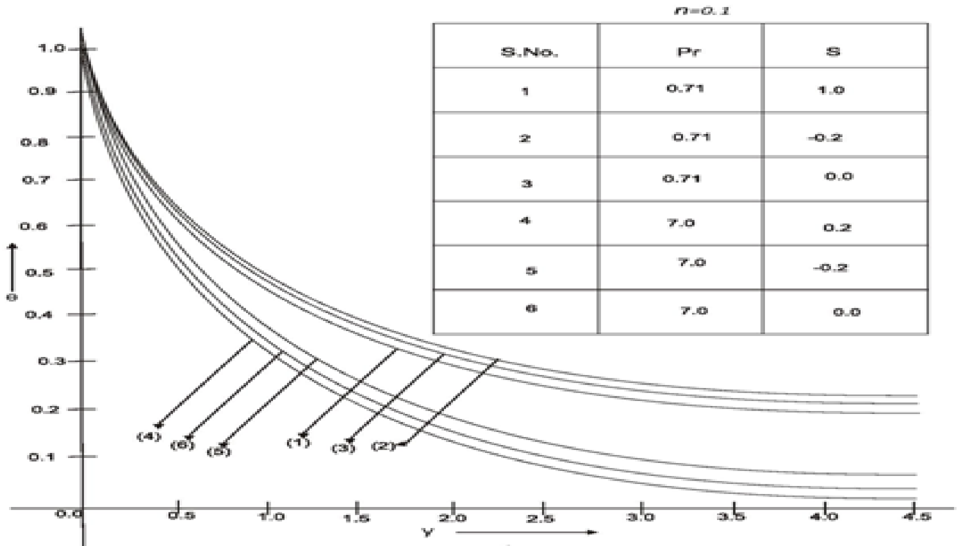
Figure 1. Velocity distribution u plotted against y for different values of K,M,α,β and h_1.

Figure 2. Angular velocity distribution ω plotted against y for different values of K, M,α,β,and h_1.



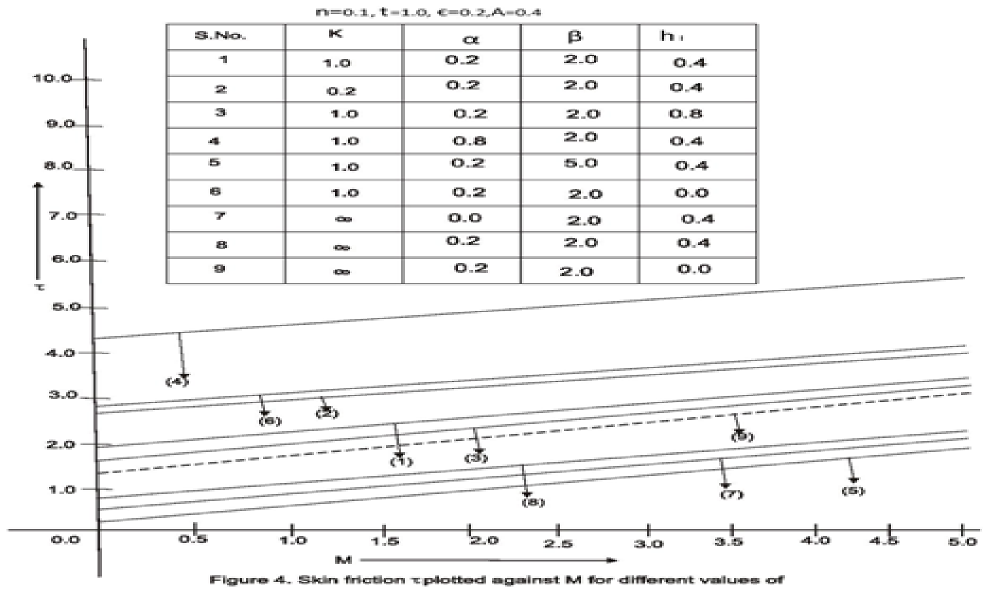
n=0.1, t=1.0, ϵ=0.2

S.No.	K	M	α	β	h_1
1	1.0	0.2	0.2	2.0	0.4
2	0.2	0.2	0.2	2.0	0.4
3	1.0	0.2	0.2	2.0	0.8
4	1.0	2.0	0.2	2.0	0.4
5	1.0	0.2	0.8	2.0	0.4
6	1.0	0.2	0.2	5.0	0.4
7	1.0	0.2	0.2	2.0	0.0
8	∞	0.0	0.2	2.0	0.4
9	∞	0.2	0.2	2.0	0.4
10	∞	0.2	0.2	2.0	0.0



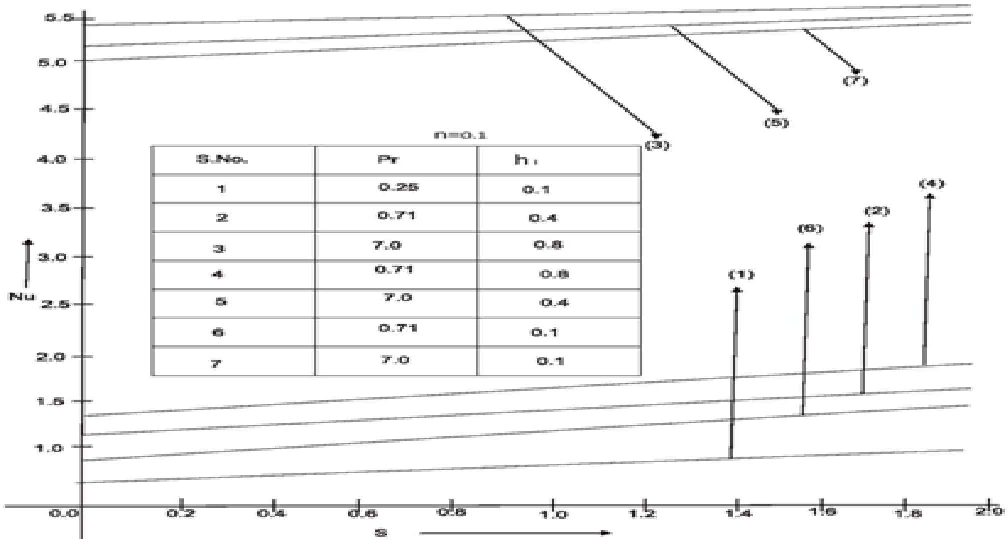
S.No.	Pr	S
1	0.71	1.0
2	0.71	-0.2
3	0.71	0.0
4	7.0	0.2
5	7.0	-0.2
6	7.0	0.0

Figure 3. Temperature distribution θ plotted against y for different values of Pr and S.



S.No.	K	α	β	h_1
1	1.0	0.2	2.0	0.4
2	0.2	0.2	2.0	0.4
3	1.0	0.2	2.0	0.8
4	1.0	0.8	2.0	0.4
5	1.0	0.2	5.0	0.4
6	1.0	0.2	2.0	0.0
7	∞	0.0	2.0	0.4
8	∞	0.2	2.0	0.4
9	∞	0.2	2.0	0.0

Figure 4. Skin friction τ plotted against M for different values of K, α , β and h_1.



S.No.	Pr	h_1
1	0.25	0.1
2	0.71	0.4
3	7.0	0.8
4	0.71	0.8
5	7.0	0.4
6	0.71	0.1
7	7.0	0.1

Figure 5.Nusselt number Nu plotted against S for different values of Pr and h_1.

V. DISCUSSIONS AND CONCLUSIONS

To explore the results, numerical values of the velocity (u), angular velocity (ω), temperature (θ), skin-friction (τ) and Nusselt number (Nu) are computed for different parameter viz. permeability parameter (K), Rotational parameter (α), Couple stress parameter (β), Magnetic field parameter (M), Prandtl number (Pr), Radiative parameter (S) and Velocity slip parameter (h_1).

In Figure 1, the velocity distribution is plotted against y for variable values of K, M, α, β and h_1 by fixing n = 0.1, t = 1.0, $\in$ = 0.2 and A = 0.4. From the figure we observed that velocity decrease with increase of K, α and β while increase with the increase of M and h_1. In case of non porous medium ($K\rightarrow\infty$) the same phenomena persist. It is further observed that when $h_1 = 0$ (No Slip Condition) velocity is decreased as compared to when $h_1 \neq 0$ it means slip velocity increase the velocity of the fluid also in case of non porous medium ($K\rightarrow\infty$) velocity increased to increase magnetic field.

In Figure 2, the angular velocity distribution is plotted against y for variable values of K, M, α, β and h_1 by fixing n=0.1, t = 1.0 and $\in$ = 0.2. From the figure we observed that angular velocity is less for higher values of α and β and more with the higher values of M, K and h_1. As expected, effects of rotational and couple stress parameters lead to fall in angular velocity.

In Figure 3, it is being calculated that temperature is more for air (Pr=0.71) in comparison with water (Pr=7.0). Moreover, increase in radiation parameter leads to fall in temperature of the fluid and negative of radiation parameter increases the temperature of the fluid.

In Figure 4, important parameter namely skin friction is plotted against M for the values of K, α, β and h_1 by taking fixed values of n = 0.1,$\in$ = 0.2, t = 1.0 and A=0.4. We conclude that more the values of K, β and h_1 less is the skin friction and on the other hand more the values of α more is the skin friction.

In Figure 5, another important parameter namely Nusselt number is plotted against S, with n=0.1 for the values of Pr and h_1. We observe that increase in Pr and h_1 increases the Nusslet number i.e. it is more for water (Pr=7.0) in comparison with air (Pr=0.71). It is observed that increase in slip parameter increases the Nusselt number for both the basic fluids air and water.

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